L_2 Similarity Metrics for Diffusion Multi-Compartment Model Images Registration

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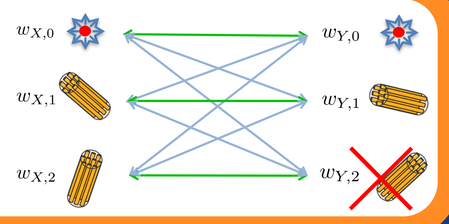
Objectives

Propose diffusion multi-compartment model (MCM) similarity measures
- Able to account for inter-patient differences
- Applicable to any diffusion MCM [1]

Go around the pairing problem
- Long: all associations tested
- Handle different numbers of compartments



Material and Methods

General MCM formulation

- PDF $p_M(x) = \sum_{i=1}^{N} w_{M,i} p_{M,i}(x)$
- CF $\varphi_M(x) = \sum_{i=1}^{N} w_{M,i} \varphi_{M,i}(x)$

Framework for pairing-free distances between MCM

- Hilbert space of square integrable functions
- Based on ℓ_2 norm between CFs

$$d^2_{\mathcal{L}}(\varphi_X, \varphi_Y) = \mathbf{w_X}^T \mathbf{A_{X,X}} \mathbf{w_X} + \mathbf{w_Y}^T \mathbf{A_{Y,Y}} \mathbf{w_Y} - 2\mathbf{w_X}^T \mathbf{A_{X,Y}} \mathbf{w_Y}$$

- $\mathbf{A}_{.,.}$: matrices of inner products of individual compartments

$$\langle f, g \rangle_{\mathcal{L}} = \int_{\mathbb{R}^3} f(x) g(x) dx$$

A simple similarity measure: SSD

$$\mathrm{SSD}_{\mathcal{L}}(\mathbf{R}, \mathbf{F}) = \sum_{k=1}^{N} d^2_{\mathcal{L}}(\varphi_{R_k}, \varphi_{F_k})$$

Accounting for patient differences: MCM correlation surrogate

- Look for "linear relationship" between CFs

$$C_{\mathcal{L}} = \min_{\boldsymbol{\theta}} \sum_{k=1}^{N} d^2_{\mathcal{L}}(\varphi_{R_k}, H_{\boldsymbol{\theta}} \varphi_{F_k})$$

- Similar to linear relationship on scalar images [2]
- Equivalent for PDF to convolution with kernel $FT(H_{\boldsymbol{\theta}})$

Handling any MCM type: spherical approximation

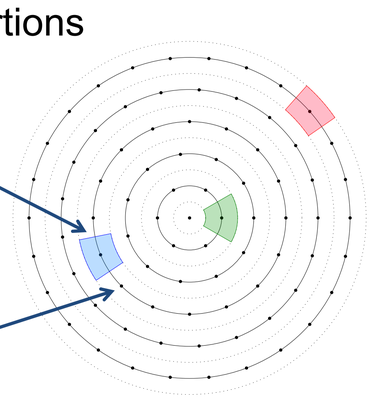
- Exploit CF relationship with DWI signal
- Numerical integration on "b-value" sphere portions

→ Approximate inner product

Volume of a subpart of a sphere

$$\langle \varphi_i, \varphi_j \rangle_{a\mathcal{L}} = \sum_k \Delta_k \varphi_i \left(\sqrt{2 b_k} g_k \right) \varphi_j \left(\sqrt{2 b_k} g_k \right)$$

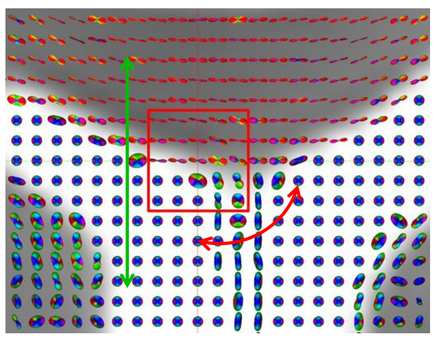
(b_k, g_k): one point on the sphere



Experiments

Experiments on HCP data
- Multi-tensor estimation, model selection [3]
- Comparison with pairing SSD as reference

Similarity behavior experiment
- Block next to the ventricles
- Translate, rotate the block

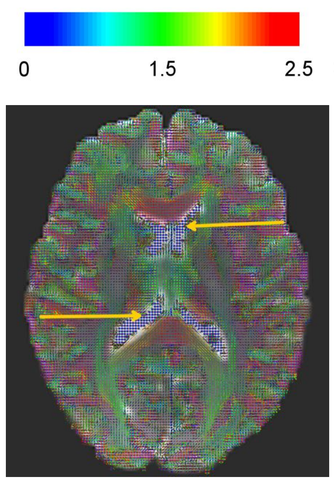


Measures included in registration framework [4], PPD re-orientation

Simulated transforms experiment
- Recover known poly-rigid transformations, measure local error

Real HCP cases experiment
- Register pairs of HCP subjects
- Measure overlap of FreeSurfer white matter parcellations

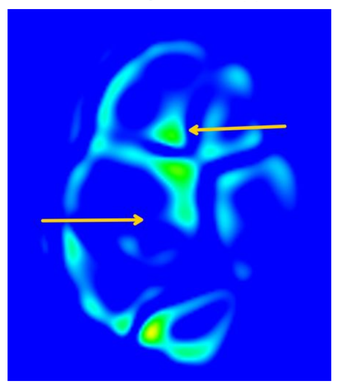
Results



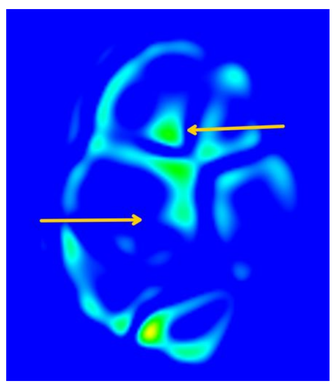
Local error scale (mm)

Curves legend:
- Approximate MCM correlation
- Approximate MCM SSD
- Pairing SSD
- MCM correlation
- MCM SSD

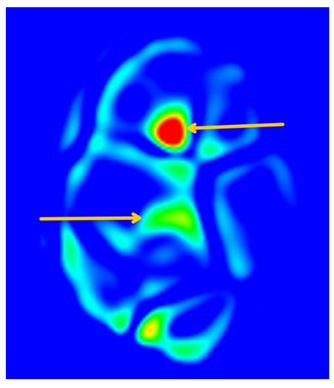
Reference image



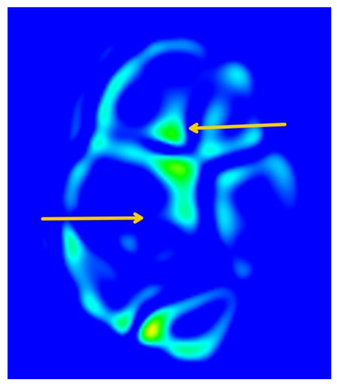
Approximate MCM SSD



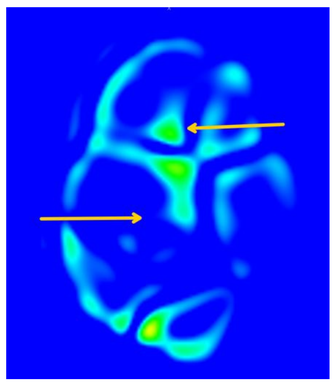
Approximate MCM correlation



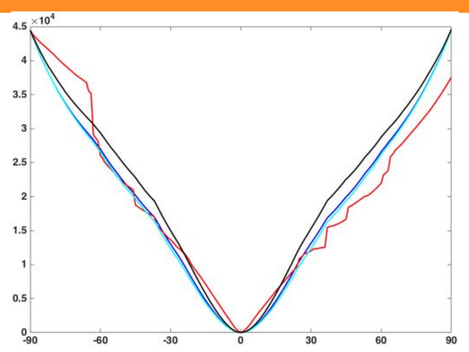
Pairing SSD



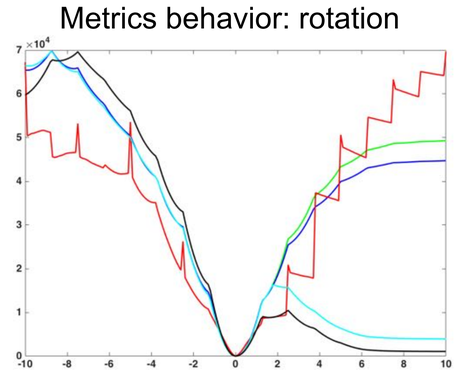
MCM SSD



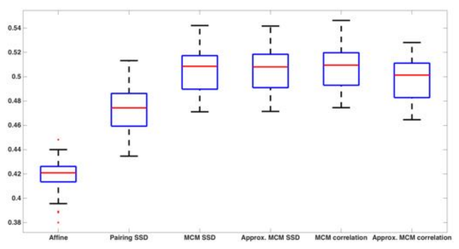
MCM correlation



Metrics behavior: rotation



Metrics behavior: translation



Overlap after HCP data registration

Summary

- Pairing free similarity metrics for MCM
  - Generic: only inner product to derive
  - Do not suffer from pairing problems
- Two metrics derived
  - SSD measure between MCM
  - MCM correlation surrogate
- Analytic derivation for multi-tensors
- Spherical approximation
  - Uses link between CF and DWI signal
- Available as part of Anima (open-source)
  - MCM interpolation and registration
  - Github link: https://goo.gl/HDC5Jf

[1] E. Panagiotaki et al. Compartment models of the diffusion MR signal in brain white matter: A taxonomy and comparison. Neuroimage. 2012.
[2] Alpert al. Improved methods for image registration. Neuroimage 1996.
[3] A. Stamm et al. Comprehensive Maximum Likelihood Estimation of Diffusion Compartment Models Towards Reliable Mapping of Brain Microstructure. MICCAI 2016.
[4] O. Commowick et al. Automated diffeomorphic registration of anatomical structures with rigid parts: application to dynamic cervical MRI. MICCAI 2012.